AMS Bologna
Internal Note
2000–01–01

Light Propagation in Curved Light Guides

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January 17, 2000

ABSTRACT

This paper presents a mathematical description of the curved light guides used by the time of flight (TOF) system proposed for the Alpha Magnetic Spectrometer in its final configuration (AMS-02). An algorithm to solve the light propagation problem into these guides is sketched.

Subject headings: AMS: TOF system — curved light guides

1. Introduction

AMS is a space experiment designed to measure the flux of the isotopes constituting Cosmic Rays [1], that will be installed on the International Space Station (ISS). In its first configuration (known as AMS-01), it was installed on the shuttle Discovery for the STS-91 mission (2 - 12 June 1998). For 10 days AMS followed an orbit similar to that of the ISS and acquired approximately 100 millions of Cosmic Rays [2].

On ISS will operate a slightly different detector, known as AMS-02, that can be viewed as an "improved" AMS-01: the Collaboration will add few subdetectors, in order to enlarge the set of measured physical quantities, but the main item is the new superconducting magnet, capable to generate a 1.5 Tesla magnetic field.

It is necessary to design a new TOF system, because the detector will be resized and because the photomultipliers used for AMS-01 (the Hamamatsu R5900U) cannot operate with this high field. In particular, in order to minimize the field influence, the PMTs will be positioned keeping their longitudinal axis nearly parallel to the magnetic field vector: a set of curved light guides will be installed.

2. The new ligth guides

The new light guides (two per counter side) will consist of two parts: a curved and bended box-like piece and the connection to the photomultiplier (rectangular to circular junction). Figure 1 shows an example of curved and bended piece. This is the worst case, because bending and twisting reach 90 degrees, and the curvature radius is 5 centimeters. Real guides will have other bending and twisting angles, between 0 and 90 degrees.

In order to understand their behaviour, a simulation program and a series of experimental tests have been set up.

3. The acceptance simulation

The mathematical description of bended and twisted voulmes is quite complicate. We considered two reference systems: the first is the "World system" (WS), with coordinates (X,Y,Z), the other is the "slice system" (SS), with coordinates (x,y,z). The SS has its (x,y) plane as slice plane, and its z axis is the tangent vector to the longitudinal axis of the guide.

We considered only continuous and constant curvatures. The longitudinal axis is bended on the (X,Z) plane of WS, and the angle is $\theta \in [0,\theta_M]$. The guide is also twisted for an angle $\phi \in [0,\phi_M]$. The angles $\theta_M \in [0,\pi/2]$ and $\phi_M \in [0,\pi/2]$ are given by the magnetic field constraints.

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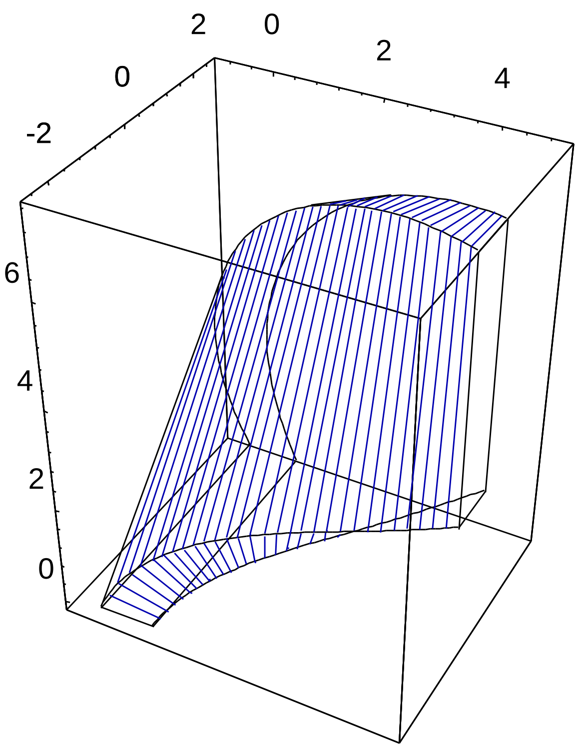

Figure 1. Light guide bended and simultaneously twisted along its axis. Values are in centimeters.

3.1. The transformation between WS and SS

We can go from WS to SS in 3 steps. First we translate the SS origin along the bended longitdinal axis, to the position (in WS):

$$\mathbf{R}_{SS} = R(1-\cos\theta, 0, \sin\theta) \tag{1}$$

where R is the curvature radius.

Second, we rotate the x axis on the (X,Z) plane, in a way that keeps it along the longitudinal axis. This rotation is described by the matrix:

$$\mathsf{A} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} . \tag{2}$$

Then we do the twisting around the z axis with the matrix:

$$\mathsf{B} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3}$$

Finally, the bending and twisting are described by the matrix:

$$\mathsf{C} = \mathsf{BA} = \begin{pmatrix} \cos\theta\cos\phi & \sin\phi & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \cos\phi & \sin\theta\sin\phi \\ \sin\theta & 0 & \cos\theta \end{pmatrix} . \tag{4}$$

The transformation then is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\phi & \sin\phi & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \cos\phi & \sin\theta\sin\phi \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \cdot \cdot \begin{pmatrix} X - R + R\cos\theta \\ Y \\ Z - \sin\theta \end{pmatrix} \tag{5}$$

while the inverse one is ($\mathsf{C}^{-1} = \tilde{\mathsf{C}}$):

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} R - R\cos\theta \\ 0 \\ \sin\theta \end{pmatrix} . \tag{6}$$

We assume that the twisting and the bending are related so that θ is the only free parameter:

$$\phi = \frac{\phi_M}{\theta_M}\theta \tag{7}$$

3.2. The simulation algorithm

Instead of consider a straight light ray propagating inside a curved volume, it's better to think about a curved light ray in a normal box.

In the WS, a straigth line is:

$$\begin{cases} X = aZ + b \\ Y = cZ + d \end{cases} \tag{8}$$

but we are interested in this light ray viewed from SS, with $z = 0$. Putting (6) (with $z = 0$) in the previous equations, and solving with respect to x and y we get:

$$\begin{cases} x = \dfrac{[1-\cos\phi(\cos\phi - c\sin\theta\sin\phi)](d+cR\sin\theta)}{\sin\phi + c\sin\theta\cos\phi} + \\ \quad + \dfrac{b - R(1-\cos\theta) + aR\sin\theta}{\cos\theta + a\sin\theta} \\ y = \cos\phi(d + cR\sin\theta) + \\ \quad - \dfrac{[b - R(1-\cos\theta) + aR\sin\theta](\sin\phi + c\sin\theta\cos\phi)}{\cos\theta + a\sin\theta} \end{cases} \tag{9}$$

For the acceptance study, the following algorithm was chosen.

1. Find $\theta_m = \min\{\theta | x = \pm x_M, y = \pm y_M\}$, i.e. put in (9) the maximum and minimum value for x and y and keep the minimum θ greater than the last recorded θ (that is initialized to 0 before to start). This θ_m will be recorded and used next time;

2. Using θ_m find the impact point $P = (X, Y, Z)$ using (6);

3. Find the normal unit vector $\hat{\mathbf{n}}$ in P to the surface of the guide. The normal direction in the SS is the line passing through $P = (x(\theta_m), y(\theta_M), 0)$ and the point Q given by $(0, y(\theta_M), 0)$ if $x(\theta_m) = x_M$, or by $(x(\theta_m), 0, 0)$ if $y(\theta_m) = y_M$;

4. Go back to the WS, and compute the reflected direction $\hat{\mathbf{v}'}$ using $\hat{\mathbf{v}} \cdot \hat{\mathbf{n}} = \hat{\mathbf{v}'} \cdot \hat{\mathbf{n}}$ and $\hat{\mathbf{v}} \times \hat{\mathbf{n}} = -\hat{\mathbf{v}'} \times \hat{\mathbf{n}}$ ($\hat{\mathbf{v}}$ is the incident ray direction). If there is total reflection, using the new direction $\hat{\mathbf{v}'}$ update the a, b, c, d parameters of eq. (8);

5. If $\theta_m \geq \theta_M$ compute the incident angle on the guide exit face to see if the ray is reflected, else return to step 1.

REFERENCES

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